CASE STUDIES OF NUMERICAL WIND ANALYSES

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ABSTRACT

With the aid of an electronic computer, case studies of wind analyses at the 850-mb., 700-mb., 500-mb., 400-mb., 300-mb., and 200-mb. pressure levels have been made. The divergent and non-divergent wind components resulting from the u and v wind-component analyses are investigated. For the cases considered, the streamfunction fields are slightly superior to the Joint Numerical Weather Prediction operational fields, obtained initially through use of the "balance equation." The magnitude of the horizontal wind divergence values are comparable to those obtained from the winds by previous investigators employing hand-analysis techniques. However, the divergence patterns are not sufficiently accurate for the strict requirements necessary for numerical weather forecasting.

1. INTRODUCTION

Analyses of several meteorological parameters have been made during the past several years at the Joint Numerical Weather Prediction Unit (JNWP) through use of electronic computer techniques. At the present time analyses of the heights of the 850-mb. and 500-mb. pressure surfaces are made twice daily on an operational, routine basis.[1] This analysis scheme was described recently by Cressman [1].

Many attempts have been made in the past to obtain accurate representations of the flow at constant pressure surfaces, but in most cases these attempts have been made through use of hand-analysis techniques. Results of studies made in recent years such as those by Landers [4], Murakami [5], Rex [7], and Taba [11] point to the possibility of obtaining direct, accurate wind analyses over regions of dense data coverage. These authors point to considerable skill in obtaining the horizontal wind divergence at several levels in the atmosphere for their individual case studies. It is a well-known fact that even with sufficient data coverage the usefulness of divergence computed from instantaneous wind observations is dependent upon a high degree of observing accuracy. Although one can hope that observational errors are random and thereby can be reduced considerably by modern analysis procedures, it is questionable whether the reduction in error will be enough to produce sufficiently accurate divergence patterns to be useful for future forecasting purposes.

The purpose of this study was to determine whether the wind fields at constant pressure surfaces in the troposphere and lower stratosphere could be accurately analyzed over the JNWP octagonal grid in an objective sense through use of a sufficiently large electronic computer.

An attempt was also made to determine to what extent, if any, the geostrophic assumption, made in the JNWP height analysis program, is damaging to the wind forecasts from the operational divergent one-parameter forecast model.

2. ANALYSIS PROCEDURE

All computations were performed by the IBM 704 electronic computer on all or part of the regularly arranged JNWP octagonal grid which covers most of the Northern Hemisphere north of 10° latitude. The grid-point interval was 381 km. at 60° latitude on a polar stereographic projection. The analysis scheme which was employed was a modification of the JNWP operational height-analysis routine which is described in detail in [1]. Only the major features of this program will be mentioned here.

This analysis technique utilizes an initial estimate of the pattern of the field being analyzed and adjusts this guess in a prescribed manner to fit the data. Therefore, for meteorological data which are sparse in certain large regions of the grid, it is desirable to use a first guess which closely approximates the final analysis over the entire region.

Attempts were made in the early stages of this study to obtain first estimates of the wind fields at all levels under consideration, excluding 500 mb., through use of a linear regression equation employing the 500-mb. streamfunction wind, obtained from the JNWP operational "balance equation" [9], and the 500–850-mb. geostrophic thermal wind. The resulting extrapolated wind fields contained large influences of the 500-mb. cyclostrophic winds which were improperly positioned. This was particularly noticeable below the 500-mb. level where the tilt with height of the pressure patterns was more pronounced.

The non-divergent part of the geostrophic wind (for further details of this see [8]) of the operational height analyses was tested for its usefulness as a first guess. The resulting analyses were not sufficiently removed from the geostrophic influences. Therefore, because of the sensitivity of the final non-divergent and, particularly,

[1] On December 15, 1960, JNWP began analyzing on an operational routine basis the height, temperature, and wind fields for 850, 700, 500, and 300 mb.

the divergent wind components to an accurate first wind-field guess, it was decided to use the balance-equation winds for the first estimates of the final analyses at all levels.

All wind data were checked in the JNWP operational automatic data processing system.[2] Four successive passes are made through the fields, and during each of these scans the following correction C to the wind component u is made at each grid point:

$$C=-WE.$$

Here, E is the error of the linearly interpolated u-wind, obtained from the first guess (or from the values computed in the previous scan), at the observation location, and

$$W=\frac{N^2-d^2}{N^2+d^2}.$$

Here, d is the distance from the observation to the grid point being modified, and N is the radius of a circle centered at the grid point. N varies from 4.75 grid intervals during the first scan to 1.0 grid intervals during the fourth scan. W is set equal to unity during the final scan. If more than one observation falls within the circle prescribed by N, a correction C is computed for each observation and the average correction is applied at the grid point under consideration.

In addition a five-point (center and four immediate surrounding grid points) smoothing operator is applied after scans 2 and 3:

$$\overline{u}=u_0+\frac{1}{8}\overline{\nabla}^2u. \tag{1}$$

Here, $\overline{u}$ denotes the smoothed value, u_0 is the value at the central point, and $\overline{\nabla}^2$ is the single grid-increment finite difference Laplacian operator.

After the last scan is completed a weak 9-point smoother, described by Shuman [10], is applied.

After the u wind component was analyzed by the above method, the procedure was repeated for the v wind component.

Because of the large barotropic 500-mb. height errors and the existence of dense upper-air data coverage over the region of interest, the cases for 0000 GMT, November 16, 1959 and 1200 GMT, December 6, 1959 were studied. Winds were analyzed over North America at the following pressure levels: 400, 300 and 200 mb. In addition, the 850-, 700-, and 500-mb. levels were analyzed over the entire JNWP octagonal grid. The scale factor of the map was used throughout this study. Thus velocities relative to the earth rather than to the map were obtained and examined. The effects of truncation errors were not considered in this study.

3. SYNOPTIC SITUATION

Since the results of the tests of both cases studied were similar, it was decided to direct most of the discussion to the 1200 GMT, December 6, 1959 case.

At this time the synoptic situation was as follows: a surface Low existed over Lake Huron with a cold front extending southward through western Florida and then southwestward through the Gulf of Mexico (fig. 1). A warm front extended eastward from this Low and then northeastward along the St. Lawrence Seaway. A weaker cyclonic circulation was centered about 450 miles east of Delaware. High pressure existed over the western United States with a ridge extending to a second high cell centered over southern Texas. A strong cyclonic circulation which had moved eastward through western Canada was centered near Ft. Nelson, British Columbia.

By 0000 GMT, December 7, 1959, the Low over Lake Huron had weakened into a trough which extended to a new cyclonic circulation centered over southeastern Virginia. Twelve hours later this storm was located in northeastern Pennsylvania and had intensified considerably—central pressure was slightly less than 980 mb. By this time the storm in western Canada had moved into the western edge of Hudson Bay and increased slightly in circulation. Aloft a strong ridge existed over the western United States with a sharp, intense trough over the Mississippi River valley at 1200 GMT, December 6, 1959 (fig. 2). This trough moved rapidly eastward during the following 24 hours.

4. EVALUATION OF RESULTS

According to Helmholtz's theorem the horizontal wind vector ($\mathbf{V}$) may be separated into a rotational non-divergent part and an irrotational divergent part; i.e.,

$$\mathbf{V}=\mathbf{k}\times\nabla\psi+\nabla\chi.$$

Here, $\mathbf{k}$ is the unit vector directed upward, ∇ is the horizontal gradient operator, and ψ and χ are the streamfunction and velocity potential, respectively. It was decided to test each of these wind components separately.

A. THE NON-DIVERGENT WIND FIELDS

From the analyzed u and v wind components it was possible to obtain the relative vorticity (ζ) from the finite difference form of the following equation:

$$\zeta=\frac{g}{f_0}\nabla^2\hat{\psi}=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \tag{2}$$

In equation (2) g is the gravitational acceleration, f_0 is the Coriolis parameter at 45° latitude, and

$$\hat{\psi}=\frac{f_0}{g}\psi.$$

Thus, $\hat{\psi}$ has dimension length. The winds were differenced over a double grid increment.

From equation (2) it was possible to obtain the field of the streamfunction, given the boundary values. These values were obtained by assigning the streamfunction of an arbitrary boundary point the value of the height of the

[2] A vertical consistency check is made on all winds greater than 15 kt. at or near the mandatory reporting levels from 1000 to 200 mb. A detailed description of this test has been given by Dent [3].

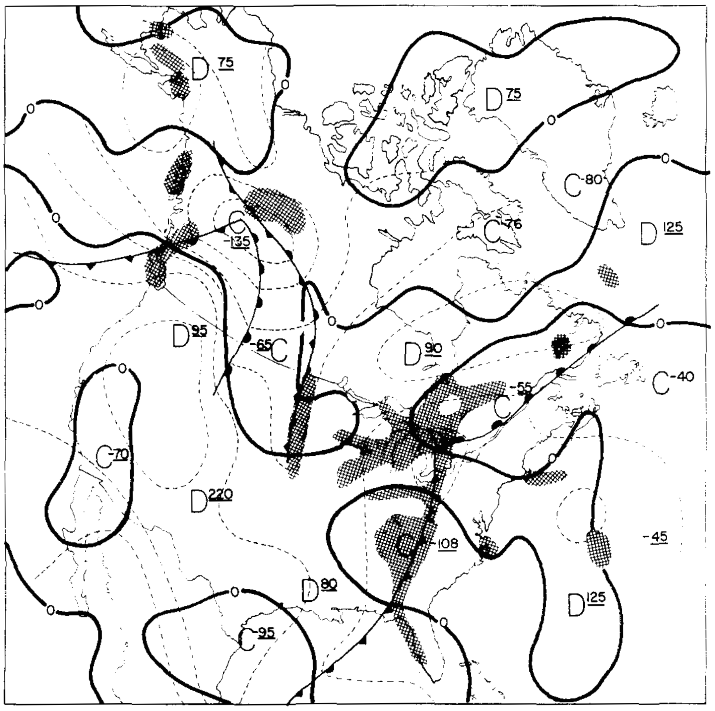

FIGURE 1.—850-mb. divergence (solid lines) and surface isobars (dashed lines) and fronts for 1200 GMT, December 6, 1959. D and C refer to divergence and convergence centers, respectively. Values are in units of 10^{-7} sec.$^{-1}$ Cross-hatching indicates precipitation areas.

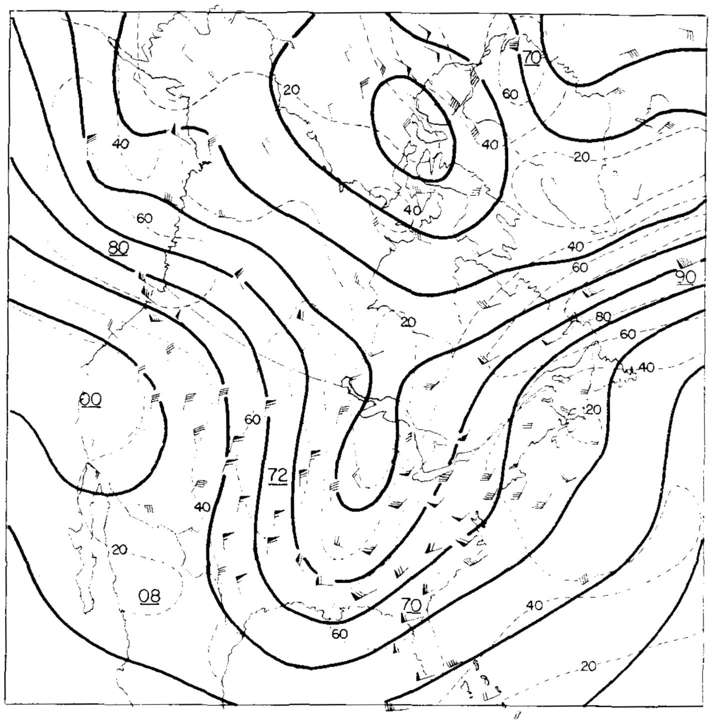

FIGURE 2.—500-mb. streamfunction lines and observed winds for 1200 GMT, December 6, 1959.

pressure surface obtained from the JNWP height analysis. From the wind components normal to the boundary it was possible to obtain a first estimate of the streamfunction at the next boundary point. Thus estimates were obtained for each boundary point. Since the integrated divergence over the region was not necessarily zero, a large discrepancy between the ψ-gradient between the first and last point and the analyzed winds at those points could result. Therefore these boundary ψ-values were adjusted, and the integrated divergence was also obtained. This was used later in obtaining the velocity potential.

The scaled streamfunction field was then obtained by the Liebmann relaxation technique using a double grid-increment finite difference Laplacian.[3] The 500-mb. result for 1200 GMT December 6, 1959 is shown in figure 2. The streamfunction isolines appear to fit the wind data quite nicely. The isotachs were computed from the analyzed winds which contained both the divergent and non-divergent components, rather than the streamfunction winds, to illustrate the accuracy of the analysis and at the same time to illustrate the effects of the analysis smoothing routines. For the 0000 GMT, November 16, 1959 case, the smoothing routines after scans 2 and 3 of the analysis procedure were omitted. Therefore, this produced analyses which were in closer agreement with the data.

The 500-mb. non-divergent wind fields obtained from this procedure were then verified over the United States and southern Canada using the reported winds. This non-divergent wind field was obtained from the streamfunction field through use of centered finite differences over a double grid-increment. The winds at the grid points were then linearly interpolated to the observation locations and compared to the observed data. The procedure was also applied to the JNWP operational streamfunction, obtained from the balance equation. The root-mean-square (RMS) and average vector errors are noted in line 2 of table 1. The same verification procedure was applied to the total analyzed wind (line 1, table 1). Here note the effects of the 2d and 3d scan smoother used in the December case.

It is not surprising that the RMS vector errors of the non-divergent wind are less than the operational errors. This is due to the fact that the operational streamfunction winds are obtained from a height analysis which utilizes

TABLE 1.—*Root-mean-square wind and height verification statistics*

500-mb. fields	0000 GMT, Nov. 16, 1959		1200 GMT Dec. 6, 1959	
	JNWP	Wind	JNWP	Wind
1. Analyzed winds (knots)	------------	5.3 (4.5)	------------	9.9 (7.8)
2. Initial non-divergent winds (knots)	15.3 *(12.2)	11.4 (9.3)	13.8 (11.3)	12.0 (9.8)
3. 12-hr. barotropic forecast winds (knots)	15.9 (13.3)	14.8 (11.8)	22.3 (18.0)	21.1 (16.3)
4. 24-hr. barotropic forecast winds (knots)	20.2 (16.9)	19.3 (16.2)	30.3 (25.3)	27.5 (22.7)
5. 36-hr. barotropic forecast winds (knots)	22.9 (19.7)	22.2 (18.8)	32.6 (27.8)	29.8 (25.4)
6. 36-hr. ψ-forecast vs. ψ-verifying winds (knots)	21.7	20.3	31.2	28.8
7. 36-hr. ψ-wind difference (knots)	43.0	------------	45.1	------------
8. Initial heights (feet)	44	81	71	105
9. 36-hr. barotropic forecast heights (feet)	348	305	423	436

*Values in parentheses are the average vector errors.

[3] A similar relaxation using a single grid-increment finite difference Laplacian produced scaled streamfunction values which differed from the double grid-increment ones by as much as 50 feet at 500 mb. in the vicinity of deep Lows.

the geostrophic approximation to fit the grid-point heights to the observed winds. In addition, these winds were used for the first guess in obtaining the wind analyses made in this study.

The 500-mb. streamfunction field obtained from the analyzed winds was further tested by making a barotropic forecast from this initial field. The JNWP operational forecast program was used for this purpose. The resulting forecast streamfunction winds were compared with those obtained from the routine barotropic forecasts issued by JNWP by the same verification procedure described above. (See lines 3, 4, and 5 of table 1.) Figures in line 6 are the 36-hr. RMS vector errors computed from the 36-hr. forecast streams and the JNWP verifying streams. A 36-hr. persistence measurement computed from the JNWP streams is given in line 7. Notice that in all forecasts the barotropic forecast winds made from the wind analyses are a slight improvement over the JNWP operational results.

The height fields were obtained from the streamfunction fields in the usual manner from the balance equation. After linear interpolation of these values to the observations, the RMS height errors, computed over the same regions as were the winds, were obtained and are presented in lines 8 and 9 of table 1. The results of the larger initial height errors from the wind analyses were surprising since it was expected that the geostropic assumption made in the JNWP height analyses was damaging. If the balance equation and its method of solution were a complete representation of the atmosphere, a streamfunction which exactly represented the observed winds should produce a height field which exactly represented the observed heights. Although the balance equation produces a frictionless, non-divergent wind and the streamfunction values at the boundary points are set equal to the height values, it is difficult to explain this discrepancy quantitatively through these approximations inherent in the balance equation and its necessary boundary restrictions. Phillips [6], while studying the Appalachian storm of November 1950, noted a similar discrepancy at 400 mb. The 36-hr. forecast-minus-verifying height pattern from the JNWP forecasts for the December case is presented in figure 3. A similar pattern was obtained from the wind-analysis forecasts.

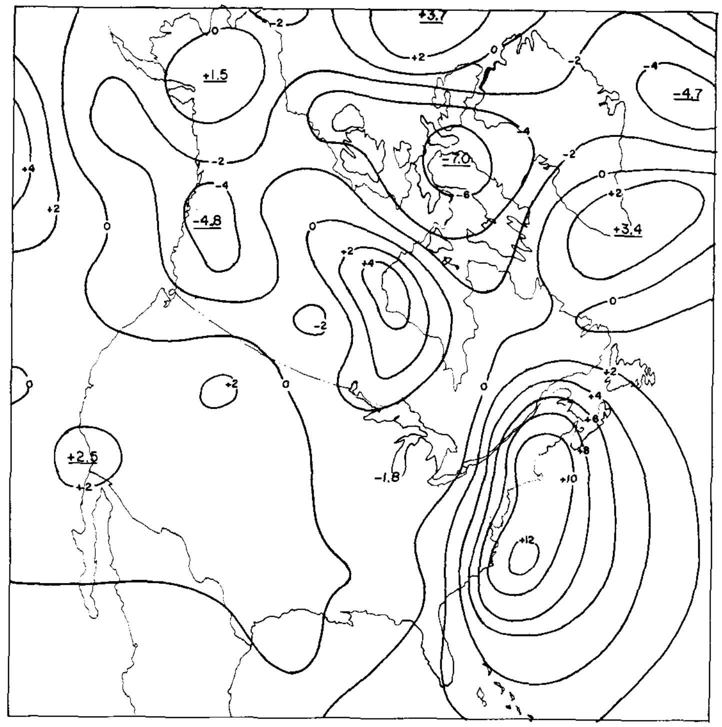

FIGURE 3.—500-mb. JNWP operational 36-hr. forecast-minus-verifying height pattern valid at 0000 GMT, December 8, 1959. Values are in units of 10^2 ft.

B. THE DIVERGENT WIND FIELDS

The fields of horizontal wind divergence were obtained directly from the u and v wind analyses at all of the levels under consideration through use of the centered finite difference approximation. The result obtained at 850 mb. for the December case is presented in figure 1. The region of convergence found over Newfoundland and southwestward over the St. Lawrence Valley is reasonable since considerable amounts of precipitation were observed in this region. Convergence was also found over the southeastern United States where cyclonic development occurred in the succeeding hours. The results at this level over the western portion of the continent are unreliable because of the presence of the mountains. The area of convergence near the southern tip of Texas is questionable.

The velocity potential field was obtained from the divergence field in the same manner that the streamfunction field was obtained from the relative vorticity field; that is, by inverting

$$\frac{g}{f_0}\nabla^2\hat{\chi}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y},$$

where

$$\hat{\chi}=\frac{f_0}{g}\chi$$

is the scaled velocity potential. A Liebmann relaxation was performed on the 500-mb. data again using a double grid-increment finite difference Laplacian. Rather than assign specific boundary values for $\hat{\chi}$ it was decided to use the average boundary normal gradient of $\hat{\chi}$ which was available from the procedure described in A above.

The result of this computation for the December case for 500 mb. is presented in figure 4a. The corresponding patterns of divergence are shown in figure 4b. The divergent wind is directed perpendicular to the isolines from centers of divergence (positive values) to centers of convergence (negative values). Over a large portion of the central United States this wind is less than 5 kt. Winds between 5 kt. and 10 kt. are found in the western and eastern United States and in southern parts of the grid. These latter features may be due, in part, to the effects of the first guess used in the wind analysis. Notice the extremely large divergent wind speeds which resulted over the south-central region of Canada. These result from the large-amplitude, short wavelength, divergence pattern obtained from the wind analysis.

C. STREAMFUNCTION TENDENCY AND VERTICAL VELOCITY COMPUTATIONS

The vorticity equation can be written in the following consistent, simplified form (see Wiin-Nielsen [12]):

$$\nabla^2 \frac{\partial \hat{\psi}}{\partial t} = -J(\hat{\psi}, \eta) - \frac{f_0^2}{g} \nabla \cdot \mathbf{V}. \tag{3}$$

Here J is the Jacobian and η is absolute vorticity. The 500-mb. scaled 12-hr. streamfunction tendency field resulting from the individual terms in the right side of the above equation and their sum are presented in figure 5 together with the "observed" tendency. The latter (fig. 5d) is an approximation obtained from the balance equation streamfunction fields 12 hr. before and 12 hr. after 1200 GMT, December 6, 1959. The 12-hr. barotropic forecast made from the wind analysis study and the operational streamfunction showed positive height errors over the eastern United States. As can be seen from figure 5a, the contribution of the first term of equation (3) produced negative stream tendencies over this region which were too small in magnitude. The magnitude and pattern of the streamfunction tendency resulting from the divergence term (fig. 5b) demonstrates that if these divergence fields were to be included in a numerical forecasting model which used these fields directly to obtain initial streamfunction tendencies, the resulting initial tendencies would undoubtedly contain large errors over regions where equation (3) was valid.

At this point it should be mentioned that another modified form of the vorticity equation was used to estimate the local streamfunction tendency patterns. Wiin-Nielsen has also shown that the relative vorticity over a large region is conserved in the following equation:

$$\nabla^2 \frac{\partial \hat{\psi}}{\partial t} = -\frac{f_0}{g} \mathbf{V}_a \cdot \nabla \eta - \frac{f_0 \eta}{g} \nabla \cdot \mathbf{V}_a. \tag{4}$$

$\mathbf{V}_a$ and η refer to the total wind and absolute vorticity, respectively. The streamfunction tendency field computed from this equation was very similar to that found using equation (3). However, the contributions of the *individual* terms could not be evaluated by the relaxation procedure because of large "pillows" of opposite sign which resulted from each of the forcing functions.

This result may be more clearly understood by considering the equation

$$\int_A \nabla^2 \frac{\partial \hat{\psi}}{\partial t} dA = -\frac{f_0}{g} \int_A \eta \nabla \cdot \mathbf{V}_a dA,$$

where A is the area. Theoretically for a sufficiently large area the above integral approaches zero *provided the absolute vorticity and the divergence are uncorrelated.* The streamfunction tendency field which resulted from the divergence term of equation (4) contained large positive errors at 500 mb. This was evident from the discrepancy

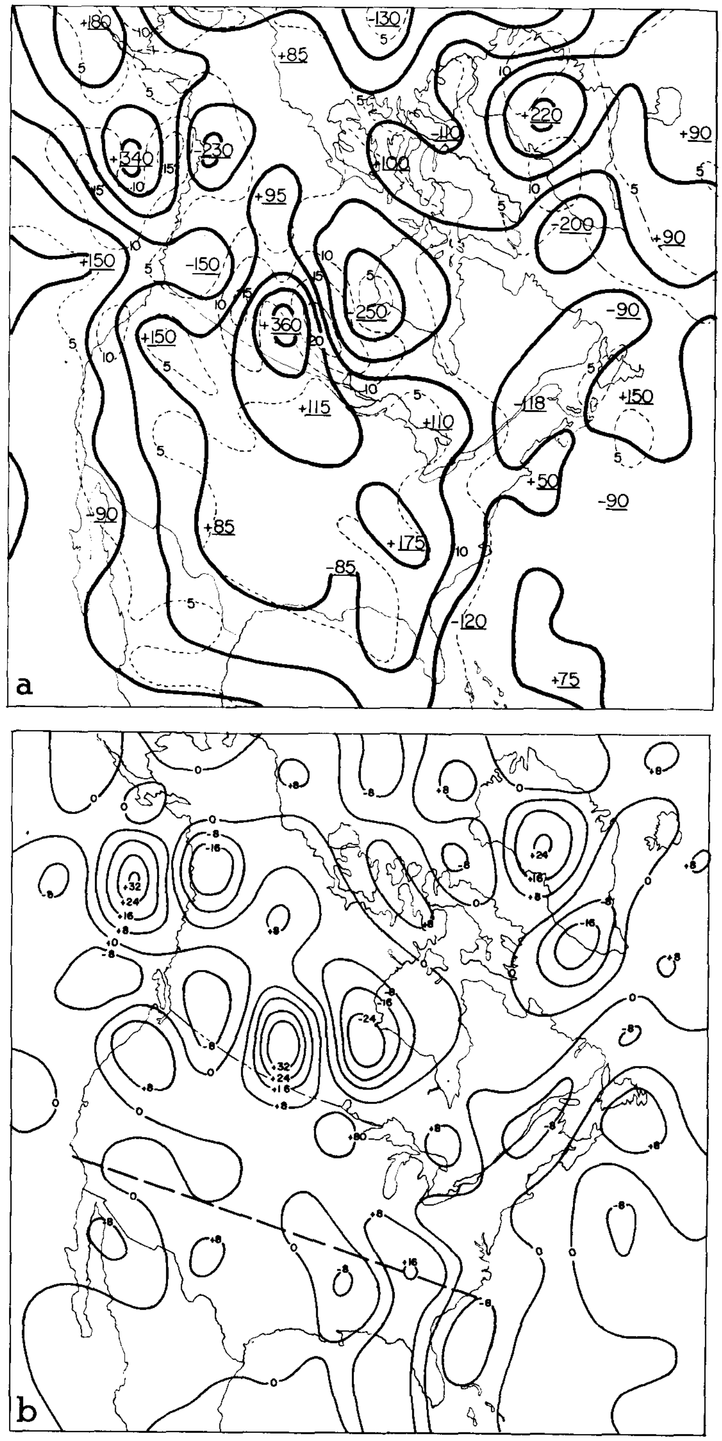

FIGURE 4.—1200 GMT, December 6, 1959. (a) 500-mb. velocity potential (solid lines) and corresponding divergent wind speed isotachs (dashed lines). Wind speeds are in knots. Plus and minus refer to centers of divergence and convergence, respectively. (b) Field of horizontal wind divergence from which (a) was obtained. Units are 10^{-7} sec.$^{-1}$ Dashed line indicates line along which vertical cross section of figure 7 was taken.

which was found in the streamfunction tendency fields resulting from the divergence terms of equations (3) and (4). Since the errors of $\frac{\partial \hat{\psi}}{\partial t}$ were positive over the central regions of the grid, the errors in $\nabla^2 \frac{\partial \hat{\psi}}{\partial t}$ were negative. Thus, according to the above equation, the vorticity and

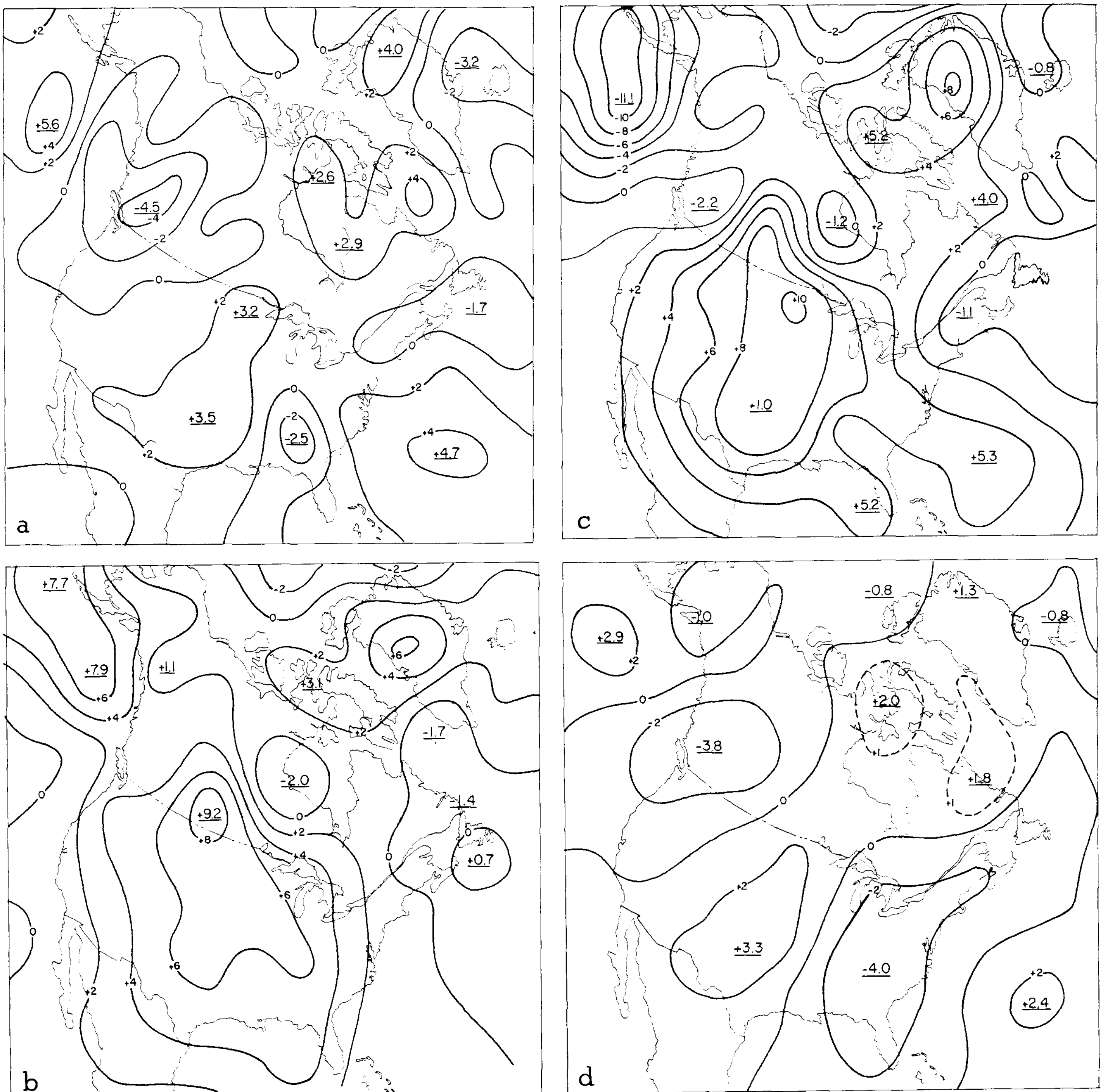

FIGURE 5.—The contribution to the 500-mb. 12-hr. scaled streamfunction tendencies for 1200 GMT, December 6, 1959, from: (a) $-J(\hat{\psi},\eta)$; (b) $-\frac{f_0^2}{g}\nabla\cdot\mathbf{V}$; and (c) $-J(\hat{\psi},\eta)-\frac{f_0^2}{g}\nabla\cdot\mathbf{V}$; (d) The streamfunction tendency obtained from operational streams 12 hr. before and 12 hr. after 1200 GMT, December 6, 1959. Units are 10^2 ft./12 hr.

divergence obtained from the wind analyses were positively correlated. In other words, at 500 mb. in regions of cyclonic vorticity horizontal divergence was prevalent, and the reverse was generally true for regions of anticyclonic vorticity.

At the 850-mb. level the individual contributions of the terms of equation (4) produced tendency pillows of the opposite sign to those found at 500 mb. Thus at these levels the vorticity and divergence were negatively correlated. Similar results were found at 700 mb.

These general results are in agreement with our present knowledge of the atmosphere. Assuming that the absolute vorticity patterns are similar throughout the lower half of the troposphere, the results imply that the level of

TABLE 2.—*Mean absolute values of divergence* (10^{-7} *sec.* $^{-1}$)

Pressure level (mb.)	Nov. 16, 1959 0000 GMT	Dec. 6, 1959 1200 GMT
850	36	43
700	35	36
500	38	47
400	46	64
300	48	63
200	43	59

least divergence over the octagonal grid exists somewhere between the 700- and 500-mb. pressure levels. This is further substantiated by the averaged divergence values computed over the United States at each of the pressure levels. (See table 2.)

A further test was applied to the divergence patterns which resulted from the wind analyses at 500 mb. If the long-wave stabilizing term and the mountain term, which exist in the JNWP operational barotropic forecast model, are neglected, the barotropic forecast equation reduces to

$$\nabla^2 \frac{\partial \hat{\psi}}{\partial t} = -J(\hat{\psi}_B, \eta_B). \tag{5}$$

The subscript B refers to the scaled barotropic streamfunction. Subtracting equation (5) from (3) produces the following approximation:

$$\nabla \cdot \mathbf{V} \approx \frac{g}{f_0^2 \Delta t} \nabla^2 \hat{\psi}_e.$$

Here $\hat{\psi}_e$ is the scaled barotropic streamfunction forecast error (forecast minus observed) and Δt is the time increment. The average wind divergence field obtained from the 24-hr. JNWP barotropic forecast from 0000 GMT, December 6, 1959 is presented in figure 6. This divergence pattern approximates the 24-hr. average pattern necessary to correct the operational JNWP barotropic forecast. The general features of this pattern may be compared with those of figure 4b, obtained directly from the wind analyses.

The approximated vertical velocity patterns were obtained from the divergence patterns (smoothed) of the analyzed winds through the use of the continuity equation:

$$\frac{\partial \omega}{\partial p} = -\nabla \cdot \mathbf{V}.$$

Here p refers to pressure and $\omega = dp/dt$. Assuming that ω is negligible at 1000 mb. and that the divergences vary linearly between the levels at which they were measured, it is possible to compute ω at all of the data levels. A cross-section of these patterns for the December case is presented in figure 7a. Figure 7b is the corresponding cross-section of the divergence fields. These cross-sections were made along a line extending from Norfolk to Los Angeles (dashed line in fig. 4b). These patterns are similar to those of the classical picture; i.e., convergence at lower levels and divergence at the upper levels appear in regions ahead of the trough and the reverse in regions to the rear of the trough. The vertical velocity errors introduced by the neglect of mountain effects are undoubtedly significant. Cross-sections of vertical velocity

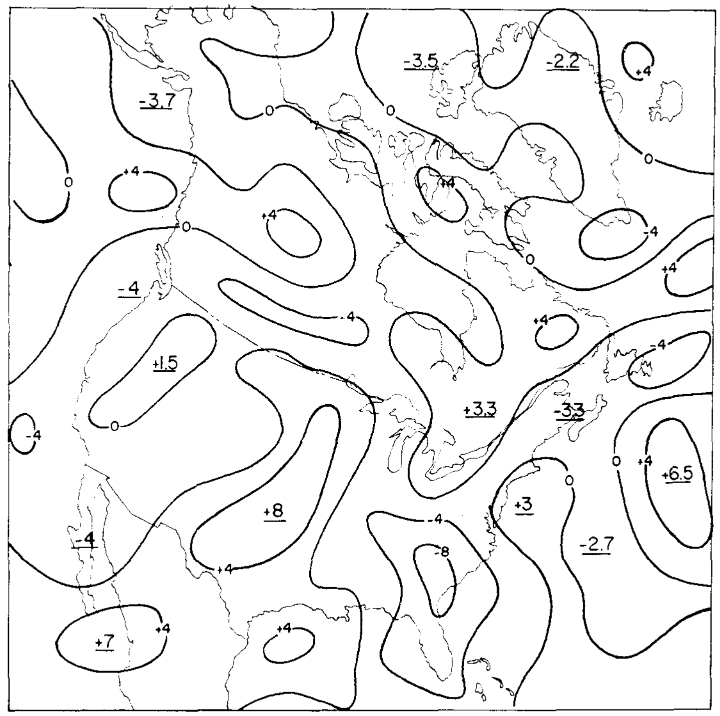

FIGURE 6.—500-mb. estimate of the horizontal wind divergence for 1200 GMT, December 6, 1959, obtained from the error field of the JNWP 24-hr. operational streamfunction forecast from 0000 GMT, December 6, 1959. Units are 10^{-6} sec.$^{-1}$

and divergence fields from Cressman's [2] three-parameter quasi-geostrophic model for this time and over the same region may be used for comparison. (See fig. 12 of Cressman's paper.) The large discrepancy found in the vertical velocities near the Continental Divide is undoubtedly due to the lower boundary restriction used in the present study. However, the large-scale features of both sets of cross-sections are in agreement in the main. The magnitudes of the divergence values are generally larger in the wind analysis study.

5. SUMMARY AND CONCLUSIONS

Although the u and v wind fields were analyzed independently, the wind fields produced by the analysis scheme "fitted" the wind observations more accurately than did the original wind fields that were used as first estimates of the final analysis. Since it was desirable to have a good first estimate of the final analysis, the JNWP operational streamfunction, obtained from the balance equation, was used for this purpose.

The streamfunction winds obtained from the wind analyses also fitted the data more accurately than did the JNWP streamfunction winds, and in both cases the former produced slightly improved barotropic wind forecasts. However, the initial height field produced from the wind-adjusted streamfunction field, which in turn was obtained through use of the balance equation program, was inferior to the operational JNWP height analysis.

The employment of the newer streamfunction in preference to the present JNWP operational one is not

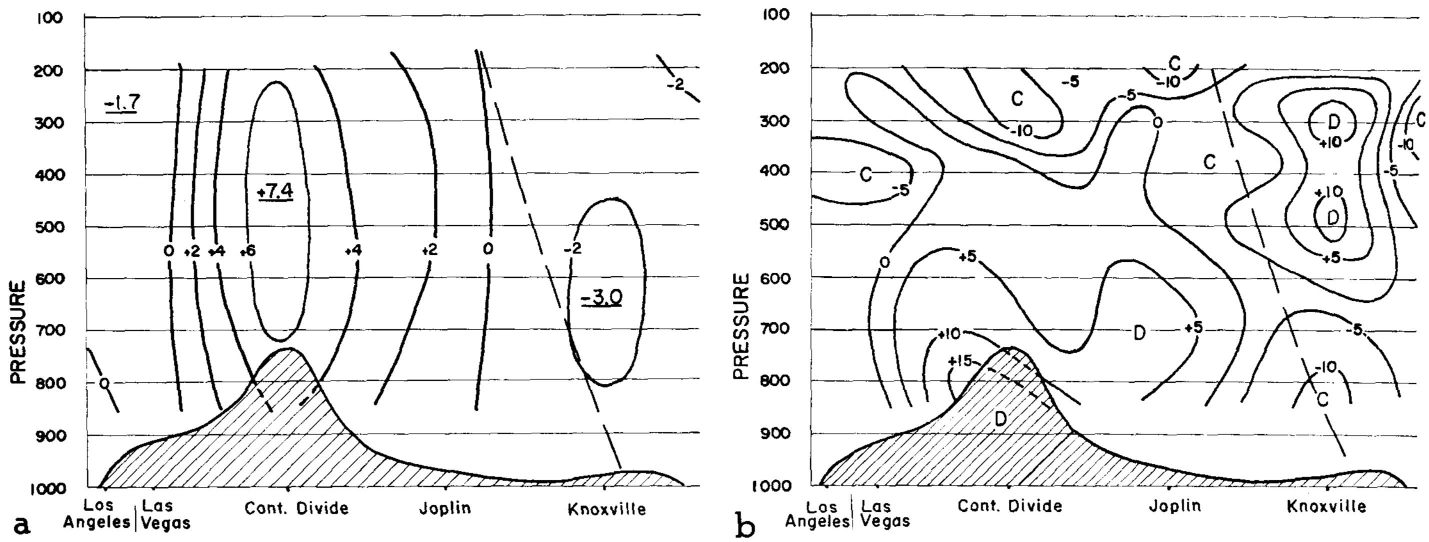

FIGURE 7.—(a) Cross-section obtained from the observed winds of 1200 GMT, December 6, 1959. Units are 10^{-3} mb. sec.$^{-1}$ (b) Corresponding horizontal wind divergence cross-section. Values are in units of 10^{-6} sec.$^{-1}$

economically feasible with the present JNWP numerical forecasting models. Preliminary tests indicate that a streamfunction of about the same quality as obtained in this study may be produced from the balance equation streamfunction by a more economical procedure.

Briefly, this entails a slight modification of the present machine analysis program to enable it to produce streamfunction analyses, using wind data only, through use of the following equations (rather than the geostrophic assumption):

$$u=-\frac{\partial\psi}{\partial y}, \qquad v=\frac{\partial\psi}{\partial x}.$$

This improved streamfunction and the one found in the present study may also prove to be more desirable, particularly in the upper troposphere and lower stratosphere, since the ellipticity criterion ($\zeta_g>-\frac{f}{2}$, where ζ_g is the geostrophic relative vorticity), which must be satisfied in solving the balance equation, may be violated in regions of dense data coverage.

The average magnitudes of the horizontal divergences resulting from the u and v wind analyses are comparable to those found by Landers and others using hand-analysis techniques. The magnitudes obtained by direct measurements appear to be larger than those obtained by most implicit calculations. As a first approximation, if we assume that the 500-mb. barotropic forecast errors are due largely to the horizontal wind divergence contribution, it would appear that the amplitudes of the divergence fields obtained in this study are of the correct order of magnitude. Although difficult to evaluate, the divergence patterns over flat terrain appear to be more accurately represented at the lower levels of the troposphere where observational density and accuracy are greater.

However, the divergence patterns obtained in this study could not be utilized by a numerical forecasting model which included the cyclogenetic mechanism and required accurate initial horizontal wind divergence measurements.

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